University of Science and Technology
Faculty of Computer Science and Information Technology
Computer Science Department
Third Year: Final Exam
Subject: Discrete Mathematics

Date 20/05/2018 **Exam Time: 3 Hours**

Answer All Questions

Question (1): (15 Marks)

1- An algorithm is a …….. of instructions solving a problem.
2- There are several properties that algorithms generally share. These properties include:…… ……., ……., and ……..
3- A …… on a set A is a subset of $A \times A$.
4- A relation R on a set A is called ..... if $(a, a) \in R$ for every element $a \in A$.
5- A relation R on a set A is called *transitive* if whenever $(a, b) \in R$ and $(b, c) \in R$, then ……, for all $a, b, c \in A$.
6- The ……. operation is used to combine two tables into one when these tables share some identical fields.
7- A graph in which each edge connects two different vertices and where no two edges connect the same pair of vertices is called a ……. graph.
8- Two vertices u and v in an undirected graph G are called …… if u and v are endpoints of an edge e of G.
9- The ……of a vertex in an undirected graph is the number of edges incident with it.
10- A …… graph on $\boldsymbol{n}$ vertices, denoted by Kn, is a simple graph that contains exactly one edge between each pair of distinct vertices.
11- A …… is a tree in which one vertex has been designated as the root and every edge is directed away from the root.
12- A tree with n vertices has…… edges.
13- An …….. in a graph G is a simple circuit containing every edge of G.
14- A simple path in a graph G that passes through every vertex exactly once is called a ……

Question (2) (19 Marks)

Solve the following problems (Algorithms and Counting)

1- Show that f (x) = x2 + 2x + 1 is $O(x^2)$. **(3 Marks)**
2- Show that n! is $O(n^n)$ and that implies log n! is O(n log n)? **(4 Marks)**
3- How many different bit strings of length seven are there? **(2 Marks)**
4- How many bit strings of length eight either start with a 1 bit ***OR*** end with the two bits 00? **(3 Marks)**
5- A multiple-choice test contains 10 questions. There are four possible answers for each question. In how many ways can a student answer the questions on the test if the student answers every question? **(3 Marks)**
6- How many strings of eight uppercase English letters are there **(4 Marks)**
   a) if letters can be repeated?
   b) if no letter can be repeated?

Question (3) (21 Marks)

Solve the following problems (Permutation and Combination)

1- In how many ways can we select three students from a group of five students to stand in line for a picture? **(3 Marks)**
2- How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school? **(3 Marks)**
3- How many bit strings of length 10 contain **(6 Marks)**
   a) exactly four 1s?
   b) at most four 1s?
   c) at least four 1s?
4- Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it **must h**ave the same number of men and women? **(3 Marks)**
5- Find the expansion of $(x + y)^6$. **(3 Marks)**
6- What is the coefficient of x^8y^9 in the expansion of $(3x + 2y)^{17}$? **(3 Marks)**

Question (4) (15 Marks)

Solve the following problems (Relations)

1- Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$, and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$. Find $S \circ R$? **(3 Marks)**
2- What do you obtain when you apply the selection operator sC, where C is the condition *Destination = Detroit*, to the database in Table 8? **(3 Marks)**

3- Display the table produced by applying the projection $P1,4$ to Table 8? **(3 Marks)**

TABLE 8 Flights.

Airline	*Flight_number*	*Gate*	*Destination*	*Departure_time*
Nadir	122	34	Detroit	08:10
Acme	221	22	Denver	08:17
Acme	122	33	Anchorage	08:22
Acme	323	34	Honolulu	08:30
Nadir	199	13	Detroit	08:47
Acme	222	22	Denver	09:10
Nadir	322	34	Detroit	09:44

4- Suppose that the relations $R1$ and $R2$ on a set A are represented by the matrices

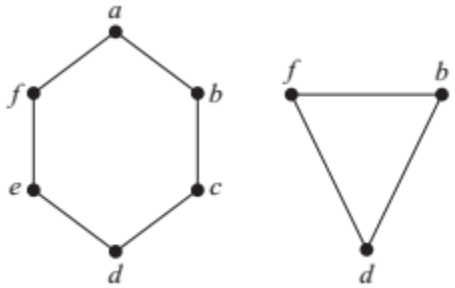
$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

What are the matrices representing $R1 \cup R2$ and $R1 \cap R2$? **(6 Marks)**

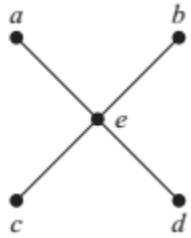
Question (5) (30 Marks)

Solve the following problems (Graphs)

1- How many edges are there in a graph with 10 vertices each vertex is of degree 6? **(2 Marks)**
2- **Draw the graphs K_3, $K_{3,3}$? (4 Marks)**
3- Find the union of the given pair of simple graphs? **(4 Marks)**



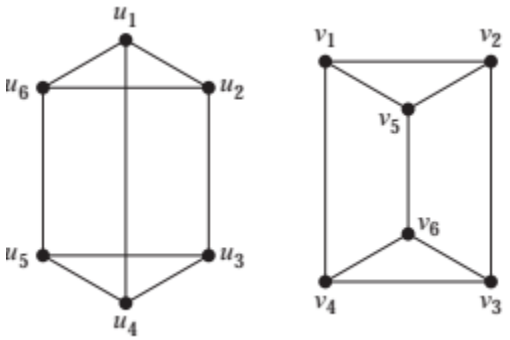
4- The complementary graph G^- of a simple graph G has the same vertices as G. Two vertices are adjacent in G^- if and only if they are not adjacent in G. Draw G^- for the following graph G? **(5 Marks)**



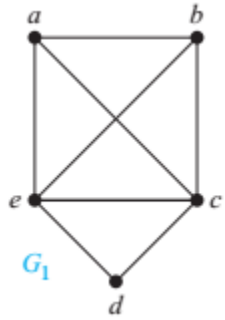
5- Draw a graph with the given adjacency matrix? **(4 Marks)**

$$\begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

6- Determine whether the given pair of graphs is isomorphic? **(5 Marks)**



7- Determine whether the given graph has an **Euler circuit, Euler Path**, **Hamilton circuit** or **Hamilton Path? (6 Marks)**



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